Position accuracy and fix rate of athletes in location monitoring

Cezary Specht1 ACD, Tomasz Szot2 ABCDEF

1 Gdynia Maritime University, Poland
2 Gdansk University of Physical Education and Sport, Poland

abstract

Background

The two main factors determining the quality of motion monitoring are the accuracy of determination of position coordinates and the frequency of position logging (fix rate).

Material/Methods

A comparative analysis of contemporary photogrammetric, remote sensing and satellite methods shows a lack of uniform requirements in this respect with reference to the same sports. Considering the issue on an intuitive basis only, it seems obvious that the accuracy of position in 100-m sprint cannot be measured in metres, and the frequency of positioning should be sub-second. However, the precise values of these variables are not estimated. A mathematical model was created which enabled the determination of minimum requirements concerning athletes’ position accuracy and fix rate, based on statistical data from sports competitions (the results from 4 Olympic Games and 6 World Championships).

Results

The key stage for this model is selecting a representative sample of 68% best results (out of a group of results) which is described by time and speed boundary values. Both variables for the selected sport (the 100-m sprint) were calculated: Mmin=0.93 m (minimum position error value) and fmin=10.88 Hz (minimum position fix rate) which enable distinguishing competitors at the finishing line (statistically, position error 5%).

Conclusions

a) The results achieved by sprinters in 100-m run in the world’s best sports events are sufficient to establish requirements regarding the accuracy and the frequency for the determination of athletes’ position in this event.

b) The statistical distribution best fitted to the population of 100-m results is the left-bounded Burr distribution (4P).

c) The method of establishing requirements for the 100-m run should be applied to other track events in order to verify an intuitive perception consisting in the lowering of accuracy and frequency requirements with an increase in an event’s distance.

Key words
human locomotion, sports, 100-m sprint
INTRODUCTION

Monitoring an athlete’s movement around a sports arena is an issue which can be considered from the navigational point of view. A change in a position understood as a change of three-dimensional coefficients (latitude, longitude and height) results in acquiring data such as distance, speed or acceleration, which facilitate their use in amateur and professional sport training.

At present, photogrammetry and remote sensing or satellite methods are used to monitor athletes’ position (Table 1). The first two methods, used at stadiums or arenas, are expensive and technically complex solutions. They require an additional process of calibration at a specialised facility, being largely dedicated to a single sport or a group of sports. These include currently used remote detection systems RedFIR (Table 1, Item 10) or photogrammetric Prozone (prozonesports.com). Satellite monitoring, where GPS or similar systems are used, has a global range; therefore, it can be used regardless of the size of a sports facility or the area in which the competition takes place. However, this requires for the competition to be held outdoors, with no terrain obstacles for signals, and for the receiver to be attached to the athlete’s body, which is banned by the regulations of some sports.

Analysing solutions applied by researchers, two key parameters of locomotion monitoring are marked out in the appended Table 1: accuracy of horizontal positioning (Column 3) and frequency of position recording (fix rate, Column 4). They both have a significant impact on acquiring kinematic quantities of athletes (distance, speed and acceleration) and on drawing further conclusions, although they are often ignored by researchers.

It seems obvious that the accuracy of position in 100-m sprint cannot be measured in metres, and the frequency of positioning should be better than 1 Hz. On the other hand, it does not seem necessary to obtain centimetre accuracy or use high frequencies while determining position in a marathon race. It must be pointed out that accuracy and fix rate of the athlete’s position can differ significantly for the same sport. With the 1998 solution, skiing was monitored with 10 Hz frequency (Table 1, entry 3), while 12 years later the volume was twice as large, i.e. 20 Hz (entry 14). Cameras were used to monitor relay races 4 x 100 m - 50 Hz, and 100-m race - 100 Hz (both entry 4). In the latter case, the goal was to compare 10-m stretches. In the analysis of 100-m sprint, a laser technology was used - 50 Hz (entry 11), as well as GPS RTK - 20 Hz (entry 14). In studies cited in the table, researchers did not mention the minimum required accuracy to be used with individual sports events. Only Supej [16], studying locomotion parameters in Alpine skiing, refers to regulations and states that if a distance between gates cannot be smaller than 6 m, a 1% error will require the accuracy of 6 cm. A natural benchmark in position frequency determination should be time measurement accuracy, which, unfortunately, is inconsistent. Table 2 shows sports that have one feature in common: competitors’ scores result from their times at the finishing line. Where the final score was the result of two or more runs, the fastest runs were taken into consideration. The table has been arranged in an increasing order in relation to the last column.
Table 1. Accuracy of position and fix rate of its logging in tools used for monitoring athletes’ locomotion on sports facilities

<table>
<thead>
<tr>
<th>#</th>
<th>Tool/system</th>
<th>Horizontal position accuracy</th>
<th>Fix rate logging used</th>
<th>Sport/event, monitoring limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 cameras with potentiometers [1]</td>
<td>-</td>
<td>2 Hz</td>
<td>soccer, camera range: pitch</td>
</tr>
<tr>
<td>2</td>
<td>1 static camera + software “Banal” [2]</td>
<td>0.26 m</td>
<td>10 Hz</td>
<td>soccer, camera range: pitch</td>
</tr>
<tr>
<td>3</td>
<td>Many cameras: TV broadcast [3]</td>
<td>-</td>
<td>10 Hz</td>
<td>skiing, giant slalom, camera range: TV broadcast</td>
</tr>
<tr>
<td>4</td>
<td>Many cameras placed perpendicularly to locomotion direction [4, 5]</td>
<td>-</td>
<td>50 Hz, 100 Hz</td>
<td>athletics, 100-, 200-, 400-m races, relay races, camera range: selected track fragments</td>
</tr>
<tr>
<td>5</td>
<td>2 static cameras + software [6]</td>
<td>0.36 m</td>
<td>-</td>
<td>handball, camera range: court</td>
</tr>
<tr>
<td>6</td>
<td>4 static cameras, system Dvideo [7]</td>
<td>0.3 m</td>
<td>7.5 Hz</td>
<td>soccer, camera range: pitch</td>
</tr>
<tr>
<td>7</td>
<td>1 static camera [8]</td>
<td>-</td>
<td>10 Hz (30 Hz) *</td>
<td>beach volleyball, camera range: court</td>
</tr>
<tr>
<td>8</td>
<td>9 cameras [9]</td>
<td>-</td>
<td>(30 Hz)</td>
<td>tennis, camera range</td>
</tr>
<tr>
<td>9</td>
<td>WASP radio system [10]</td>
<td>-</td>
<td>10 Hz (125 Hz)</td>
<td>wheelchair rugby, radio signal range: court</td>
</tr>
<tr>
<td>10</td>
<td>RedFiR radio system [11]</td>
<td>a few cm</td>
<td>(200 Hz)</td>
<td>soccer, radio signal range: pitch</td>
</tr>
<tr>
<td>11</td>
<td>Laser Jenoptik LavegSPORT [12]</td>
<td>-</td>
<td>50 Hz</td>
<td>athletics, 100-m sprint, device range: the straight, parallel to motion direction</td>
</tr>
<tr>
<td>12</td>
<td>DGPS [13]</td>
<td>-</td>
<td>0.5 Hz</td>
<td>foot orienteering, no limitations at the competition area</td>
</tr>
<tr>
<td>13</td>
<td>GPS RTK [14, 15]</td>
<td>-</td>
<td>up to 20 Hz</td>
<td>walking (not race walking), no limitations at the area under survey</td>
</tr>
<tr>
<td>14</td>
<td>GPS RTK [16, 17]</td>
<td>5-10 mm</td>
<td>20 Hz</td>
<td>downhill skiing, 100 m sprint, no limitations at the area under survey</td>
</tr>
<tr>
<td>15</td>
<td>GPS Catapult MinimaxX [18]</td>
<td>-</td>
<td>5 Hz</td>
<td>beach soccer, no limitations at game area</td>
</tr>
<tr>
<td>16</td>
<td>GPS [19]</td>
<td>-</td>
<td>4 Hz</td>
<td>rowing, no limitations at the competition area</td>
</tr>
</tbody>
</table>

* - no data available ; * Cut-off frequency of the system in brackets

Table 2. Distance in the smallest unit of time measurement in selected sports/events

<table>
<thead>
<tr>
<th>#</th>
<th>Sport/event</th>
<th>Distance (m)</th>
<th>Time (s)</th>
<th>Mean speed (m/s)</th>
<th>Time measurement accuracy (1/…s)</th>
<th>Distance covered in the smallest unit of time accuracy (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>mountain biking, downhills M (MTB WC 2012 Leogang)</td>
<td>2600</td>
<td>201.790</td>
<td>12.884</td>
<td>1000</td>
<td>0.012</td>
</tr>
<tr>
<td>2</td>
<td>motorsport, race (GSMP Sopot 2012)</td>
<td>3050</td>
<td>86.649</td>
<td>35.199</td>
<td>1000</td>
<td>0.035</td>
</tr>
<tr>
<td>3</td>
<td>motorsport, F1 GP (Australia 2012)</td>
<td>307574</td>
<td>5649.565</td>
<td>54.442</td>
<td>1000</td>
<td>0.054</td>
</tr>
<tr>
<td>4</td>
<td>athletics, 400 m sprint M (OG London 2012)</td>
<td>400</td>
<td>43.94</td>
<td>9.103</td>
<td>100</td>
<td>0.091</td>
</tr>
<tr>
<td>5</td>
<td>athletics, 100 m sprint M (OG London 2012)</td>
<td>100</td>
<td>9.63</td>
<td>10.384</td>
<td>100</td>
<td>0.103</td>
</tr>
<tr>
<td>6</td>
<td>skiing, giant slalom M (FIS WC Adelboden 2013)</td>
<td>1290</td>
<td>74.06</td>
<td>17.418</td>
<td>100</td>
<td>0.174</td>
</tr>
<tr>
<td>7</td>
<td>skiing, downhill M (FIS WC Kitzbuhel 2013)</td>
<td>3312</td>
<td>117.56</td>
<td>28.172</td>
<td>100</td>
<td>0.281</td>
</tr>
<tr>
<td>8</td>
<td>athletics, racewalking 50 km M (OG London 2012)</td>
<td>50000</td>
<td>12959</td>
<td>3.858</td>
<td>1</td>
<td>3.858</td>
</tr>
<tr>
<td>9</td>
<td>athletics, marathon M (OG London 2012)</td>
<td>42195</td>
<td>7681</td>
<td>5.493</td>
<td>1</td>
<td>5.493</td>
</tr>
<tr>
<td>10</td>
<td>bicycle race (Paris-Roubaix 2012)</td>
<td>257500</td>
<td>21322</td>
<td>12.076</td>
<td>1</td>
<td>12.076</td>
</tr>
<tr>
<td>11</td>
<td>bicycle race, time trial (prologue, Tour de France 2012)</td>
<td>6400</td>
<td>433</td>
<td>14.781</td>
<td>1</td>
<td>14.781</td>
</tr>
</tbody>
</table>

M – men, MTB – mountain bike, WC – World Cup, GSMP – Uphill Race Polish Championship, OG – Olympic Games, FIS – International Ski Federation
Some inconsistency can be seen in Table 2. Within the same time unit (1 Hz), a racewalker covers 3.8 m, and a cyclist in a time trial – 14.8 m (entries 8-11). In cycling, a downhill race (Item 1) is measured with a 1,000 Hz accuracy, and time trial (Item 11) a thousand times less frequently, although the achieved average velocities are different by merely 1 m/s (several per cent). Therefore, some methods other than those based on time measurement accuracy should be used to determine the accuracy and frequency of position.

**RESEARCH QUESTIONS AND OBJECTIVES**

The diverse nature of sports, including a diversity of locomotion forms, makes it necessary to narrow down the problem. A run in a straight line is the simplest motion to describe; therefore, the objective of this article is to determine minimum requirements for the accuracy and frequency of an athlete’s position during 100-m sprint.

The following research questions were formulated:

What criterion should be adopted in the process of mathematical modelling to establish minimum requirements for the determination of the accuracy and the fix rate of an athlete’s position in sports monitoring?

Is it possible to determine the minimum accuracy and frequency requirements on the basis of results achieved by runners in 100-m sprint?

What are statistical characteristics of the result population in 100-m sprint?

Can the method of establishing requirements for 100-m sprint be used in other track races?

**MATERIALS AND METHODS**

The subject of the research was final runs (M) during the Olympic Games (2000–2012) and the World Championships (2001–2011), obtained from the result database of IAAF (International Association of Athletic Federations, iaaf.org). The authors selected those competitions due to their high sports standard, and the male category was chosen because results achieved by men are better than those achieved by women (Table 3). Competitors who failed to complete their runs were not included.

Table 3. The 100-m man final run results of Olympic Games and World Championships (2000–2012)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.99</td>
<td>9.94</td>
<td>10.08</td>
<td>9.86</td>
<td>10.05</td>
<td>9.91</td>
<td>9.89</td>
<td>9.71</td>
<td>10.08</td>
<td>9.75</td>
</tr>
<tr>
<td>3</td>
<td>10.04</td>
<td>9.98</td>
<td>10.08</td>
<td>9.87</td>
<td>10.05</td>
<td>9.96</td>
<td>9.91</td>
<td>9.84</td>
<td>10.09</td>
<td>9.79</td>
</tr>
<tr>
<td>5</td>
<td>10.09</td>
<td>10.07</td>
<td>10.21</td>
<td>9.94</td>
<td>10.09</td>
<td>10.08</td>
<td>9.95</td>
<td>9.93</td>
<td>10.26</td>
<td>9.88</td>
</tr>
<tr>
<td>6</td>
<td>10.13</td>
<td>10.11</td>
<td>10.22</td>
<td>10.00</td>
<td>10.13</td>
<td>10.14</td>
<td>9.97</td>
<td>10.00</td>
<td>10.27</td>
<td>9.94</td>
</tr>
<tr>
<td>7</td>
<td>10.17</td>
<td>10.24</td>
<td>-</td>
<td>10.10</td>
<td>10.14</td>
<td>10.23</td>
<td>10.01</td>
<td>10.00</td>
<td>10.95</td>
<td>9.98</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10.20</td>
<td>10.29</td>
<td>10.03</td>
<td>10.34</td>
<td>-</td>
<td>11.99</td>
<td></td>
</tr>
</tbody>
</table>
In further analyses, the authors created the empirical distributions defined by the function:

\[ F(x_i) = P(W_k < w), \ w \in R \]  \hspace{1cm} (1)

where:

\( F(x_i) \) - empirical distribution function of the competition \( i \),

\( W_k \) - result of a competitor \( k \),

for variable \( i \), which in the function of competitions may have 2 values:

\( i = \text{OG2000, OG2004, OG2008, OG2012 - Olympic Games,} \)


For comparative purposes, a cumulative distribution function (including all 10 competitions WC & IO in question) was defined. This enables relating results to expected values based on a statistical analysis of numerous competitions. Fig. 1 and Fig. 2 show the above-said distribution functions, with a division of competitions into the Olympic Games and the World Championships.

The graphs show that the results in the Olympic Games (Fig. 2) are significantly better than the mean of the aggregate population – almost all the curves of Olympic Games can be found at the left-hand-side of the cumulative distribution function. A high level of the 2009 World Championships is also noteworthy (Fig. 1, WC2009).

For the purpose of the assessment of statistical distribution of aggregate results, in order to fit the best distribution, random variable testing was carried out with the use of “EasyFit 5.5 Professional” software. The best fittings were demonstrated by the 4-parameter Burr distribution (for parameters: \( k = 0.80545, \alpha = 8.4007, \beta = 0.79676, \gamma = 9.1793 \); where \( k \) and \( \alpha \) - continuous shape parameters, \( \beta \) - continuous scale parameter, \( \gamma \) - continuous location parameter), followed by Log-Logistic, Log-Logistic (3P) and Burr (3P) distributions. The Burr distribution is applied, among others, in research on household revenues, insurance risks, and reliability analysis [20, 21]. Fig. 3 shows the probability density function for parameters circumscribed on the empirical variable.

Then, the empirical distribution function was juxtaposed with the Burr (4P) distribution standardised values (Fig. 4). Knowing the distribution parameters, we can determine any probability measures based on the probability density function \( g(x) \).
Fig. 1. Empirical distribution functions of the results of the final 100-m in the World Championships (2001–2011) and the cumulative distribution function (in bold gray)

Fig. 2. Empirical distribution functions of the results of the final 100-m sprint in the Olympic Games (2000–2012) and the cumulative distribution function (in bold gray)
In order to formulate a functional model of minimum requirements regarding the accuracy and the fix rate of positioning athletes’ location, it was necessary to establish a uniform criterion. The criterion was based on the assumption that a minimum accuracy and fix rate of athletes’ position should enable the determination of the order in which they reach the finishing line. The method consists of three stages:

Stage I. Selecting a representative sample of 68% best results out of a group of results (10 OG and WC finals),
Stage II. Determining a minimum position fix rate, which enables distinguishing competitors at the finishing line (statistically),
Stage III. Determining a minimum position accuracy, which enables distinguishing competitors at the finishing line (statistically).
STAGE I

In order to exclude outlying results (statistically) from analyses, it was reasonable to assume a specified, statistically representative population of results under analysis. Therefore, it was decided that 68% of the sample of the best results would be submitted to further analyses (although, as it has been proved above, a probability distribution for 100 m run is not a normal distribution, the authors decided to adopt this value in line with the three-sigma rule). As mentioned before, in order to determine a limit result - the minimum result above which 68% of competitors scored better - a population was sorted out (74 results, a set of 10 recent OG and WC competitions). This analysis provided the 10.08 s value, which means that 68% of the population achieved better scores. The idea of this stage is shown in Fig. 5. Competitors whose times do not fall into the interval in question are marked light grey (Fig. 5, on the left, below 10.08 s). The results of the representative group are shown in the form of the distribution function (Fig. 6).

Fig. 5. Exclusion of a group of outlying results for the assessment of a representative population
STAGE II

For a single race with 8 participants, the population under research is 5.44 competitors (8 people x 68% = 5.44 people) who achieve times of 9.58 s - 10.08 s. In order to meet the predefined requirement of distinguishability (statistically) at the finishing line, it must be assumed that 5.44 competitors (68% of the population of a single race) will across the finishing line in the interval being the difference between a minimum and maximum times. Therefore, their distinguishability (statistically) in time will be:

\[ R = \frac{t_{\text{min}(68\%)} - t_{\text{max}(68\%)}}{L \cdot 0.68} = \frac{10.08 - 9.58}{8 \cdot 0.68} = 0.09191 \text{s} \]  

(2)

where:

- \( t_{\text{max}(68\%)} \) - the maximum time in the population of 68% of best results in sports events under analysis,
- \( t_{\text{min}(68\%)} \) - the minimum time in the population of 68% of best results in sports events under analysis,
- \( L \) - number of competitors in a single final (for 100-m run – 8 people).

A minimum position fix rate will be determined as the converse of distinguishability:

\[ f_{\text{min}} = \frac{1}{R} = 10.88 \text{Hz} \]  

(3)
STAGE III

In order to determine a minimum position accuracy, let us assume a position error requirement of 5%, commonly used in maritime, air and land navigation, described as $p = 0.95$. This means a radius of the circle which includes 95% of measurements. For example, for the GPS system, it is 9 m horizontally and 15 m vertically [22, p. 34]. In order to determine a minimum accuracy of the position fix rate, a minimum speed in the 68% block (time 10.08 s) is 9.9206 m/s, and a maximum (time 9.58 s) is 10.4384 m/s, thus the mean distance between competitors can be calculated statistically by multiplying it by distinguishability:

$$R = \frac{V_{\text{max}(68\%) + V_{\text{min}(68\%)}}}{2} \cdot 0.09191 \text{s} = 0.9356 \text{m}$$  \hspace{1cm} (4)

where:

$M_{\text{min}}(p = 0.95)$ – minimum position error value of a competitor’s position with 0.95 probability,

$V_{\text{max}(68\%)}$ – maximum speed in the population of 68% best results in sports events under analysis,

$V_{\text{min}(68\%)}$ – minimum speed in the population of 68% best results in sports events under analysis.

These calculations show that the minimum accuracy of monitoring competitors’ position in 100-m run is 0.93 m, which enables (statistically) their differentiation at the moment they come across the finishing line.

DISCUSSION

The starting point for this article was drawing attention to the lack of unified requirements concerning equipment used for monitoring athletes’ locomotion in sports (as discussed in the Introduction, and synthetically presented in Table 1). Since acquiring numerical values of locomotion results from position changes in the function of time, it was indicated that the key requirements to be considered are accuracy and frequency of determining athlete’s position. These requirements were related to the simplest form of locomotion, i.e. sprint along a straight track, where time becomes a result of final classification. After analyzing 100 m sprint finals during the World Championships and Olympic Games, it was noticed that a minimum accuracy an athlete should be monitored with is $M_{\text{min}} = 0.93 \text{m}$, and a minimum frequency of establishing his position should be $f_{\text{min}} = 10.88 \text{Hz}$ (both parameters allow for doing it with a probability level of 95%). The calculated values refer to a statistical population of results and an attempt to use them to determine the order of runners at the finishing line may not be satisfying (e.g. the 2003 World Championships – differences of 0.01 s or no differences at all), but this is not what the presented model is meant for. It must be stressed here that the established values are not designed to determine competitors’ position at the finishing line,
but they constitute minimum requirements for monitoring a competitor’s (or a group of competitors) location for training purposes of very specific standards. Depending on how well athletes are trained, we will obtain different requirements for lower-level competitions (lower accuracy and lower fix rate).

The mathematical model for the determination of minimum accuracy and fix rate of athletes’ location presented here has been verified in other stadium races and is going to be discussed in further publications.

CONCLUSIONS

(a) The results achieved by sprinters in 100-m run in the world’s best sports events are sufficient to establish requirements regarding the accuracy and the frequency for the determination of athletes’ position in this event.

(b) The statistical distribution best fitted to the population of 100-m results is the left-bounded Burr distribution (4P).

(c) The method of establishing requirements for the 100-m run should be applied to other track events in order to verify an intuitive perception consisting in the lowering of accuracy and frequency requirements with an increase in an event’s distance.

REFERENCES


